TITLE OF COURSE: Ordered vector spaces and population dynamics

INSTRUCTOR: Horst Thieme

TIME: MW 3:05-4:20

LOCATION: Tempe - ECG G317

APM 598 CLASS #: 27190

MAT 598 CLASS #: 31751

The reals numbers carry two important structures in addition to the field structure: a topology induced by the absolute value and a partial order. In this special case, these two structures basically amount to the same because one can be expressed in terms of the other.
As for multivariable analysis, the usual courses only generalize the topological structure by considering metric or, more generally, topological spaces. Function spaces, however, have both a topology and a natural partial order, the pointwise order, where one can no longer replace the other. Rather, these two structures interact in a fruitful way. In vector spaces, partial orders are typically linked to cones (in function spaces the set of nonnegative functions). For many applications in the social, biological, and economic sciences, it is natural to look for solutions in cones because population densities and the like are nonnegative. Biological, social and economic dynamics, if modeled in discrete time units, can be described by iterating maps acting on cones (discrete dynamical systems). The persistence or extinction of a population typically depends on the spectral radius of approximations of such maps at the origin (the extinction state). For population models that ignore the existence of males and females, this approximation is linear, but otherwise the approximation is only positively homogeneous. This course offers a rigorous introduction into these concepts which just have been informally described.

Prerequisites: Knowledge of Linear Algebra, Multi-variable Calculus, and the basics of metric spaces and normed vector spaces.

Requirements: weekly homework

Detailed course notes will be distributed as PDF files via email.

Optional background reading: Bas Lemmens, Roger D. Nussbaum, Nonlinear Perron-Frobenius Theory, Cambridge University Press, 2012