MAT 573 - Second Course in Complex Analysis
Instructor - Al Boggess
Spring 2018 - MW 3-4:20 pm

The Goal of this second course is to hit a few highlight topics in complex analysis not covered in MAT 572 (which is the prerequisite for this course). These topics will be presented in lecture. The other goal is for each participant to choose a topic related to complex analysis for further study and report. Suggestions are given below.

Course Grade will be determined by an oral and written report on the chosen project. There will be no graded homework, quizzes or exams.

Outline of Topics Covered in Lecture.

1. *The Riemann Mapping Theorem* - which states that any simply connected region in the plane which is not the entire plane is analytically equivalent to the unit disc. Interestingly, this theorem only holds for domains in $\mathbb{C}^1$ and not for higher dimensions. Techniques involve normal families and the Arzela Ascoli Theorem

2. *Runge's Approximation Theorem* - states that analytic functions on a domain $\Omega \subset \mathbb{C}$ can be approximated uniformly on compact subsets of Ω by entire functions if and only if Ω has no "holes" (i.e. the complement of Ω is connected). The classic "pole-pushing" argument will be given for its proof.

3. *Solving the inhomogeneous Cauchy-Riemann equations*: $\dfrac{\partial f}{\partial \bar{z}} = g$ on a domain $\Omega \subset \mathbb{C}$, where g is a given smooth function on Ω and f is the sought-after solution. When $\Omega = \mathbb{C}$ and g has compact support, a nice neat convolution formula with the Cauchy kernel will do the trick. The more general case will involve piecing together the solutions in the case where the right side is compactly supported together with the use of Runge's approximation theorem (as mentioned above).

4. *Consequences* of solving the inhomogeneous Cauchy-Riemann equations, including Mittag Leffler's Theorem on prescribing principal parts of Laurent Expansions, and an interpolation theorem.

List of Possible Project Topics. There are many projects for students. Here are a few examples:

1. *The Schwarz Christoffel Transformation* used in mapping a polygonal region to the unit disc.

2. *Infinite Products* how are they defined and what are their basic properties, and their use in certain special functions, such as the Gamma Function.

3. *Non Euclidean Geometry* and its relation to complex analysis

4. *Topics in Harmonic Functions* and Applications.

5. *The Fourier Transform* is one of the key tools used in partial differential equations. It turns out that the Fourier transform has an inverse which equals its L^2 adjoint and this remarkable property is one of the reasons it is such a valuable tool. Some complex analysis is commonly used in the establishment of this result.

6. *The Prime Number Theorem.* - Loosely speaking, this result states that the proportion of primes which are less than or equal to x is close to the fraction $1/\ln x$ when x is large. The proof requires a lot more complex analysis than the algebra of numbers

Students are welcome to suggest their own ideas for projects (with instructor approval).