Name _Solution_   Student ID No. _____

Directions:

- There are 5 multiple choice questions worth 7 points each, 4 true or false questions worth 4 points each, and 2 free response questions.

- You must show your work on all questions.

- You must give a clear and correct explanation or provide a counter-example for the true/false questions to receive credit.

- Partial credit is only available on the free response problems.

- Read all the questions carefully. No calculators with QWERTY keyboards or ones like TI-89 or TI-92 that do symbolic algebra may be used.

- Put the final answer to the space provided in an orderly fashion. Box your final answers. No partial credit will be given if more than one answer is given, or if it unclear which answer is meant to be your final answer.

Honor Statement

By signing below you confirm that you have neither given nor received any unauthorized assistance on this exam. This includes any use of a graphing calculator beyond those uses specifically authorized by the School of Mathematical and Statistical Sciences and your instructor. Furthermore, you agree not to discuss this exam with anyone in any section of MAT 272 until the exam testing period is over. In addition, your calculator's program memory and menus may be checked at any time and cleared by any exam proctor.

Signature:

Date:

7 points each (handwritten margin note)

1. The equation of the sphere with center $(2, -3, 6)$ that touches the yz-plane is

    (a) $(x - 2)^2 + (y + 3)^2 + (z - 6)^2 - 4 = 0$ *(selected)*

    (b) $(x - 2)^2 + (y + 3)^2 + (z - 6)^2 - 6 = 0$

    (c) $(x + 2)^2 + (y - 3)^2 + (z + 6)^2 = 4$

    (d) $(x - 2)^2 + (y + 3)^2 + (z - 6)^2 = 2$

    (e) $(x + 2)^2 + (y - 3)^2 + (z + 6)^2 = 36$

    (f) Not enough information or none of the above

    Handwritten: $(x-2)^2 + (y+3)^2 + (z-6)^2 = 2^2$

2. The area of the triangle with vertices $P(1, -1, 0)$, $Q(2, 1, -1)$ and $R(-1, 1, 2)$ is

    (a) $3\sqrt{2}$ *(selected)*

    (b) 4

    (c) 8

    (d) $\sqrt{34}$

    (e) Not enough information or none of the above

    Handwritten:
    $\vec{PQ} = \langle 1, 2, -1 \rangle$
    $\vec{PR} = \langle -2, 2, 2 \rangle$
    $\vec{PQ} \times \vec{PR} = \langle 4+2, -(2-2), 2+4 \rangle$
    $= \langle 6, 0, 6 \rangle$
    $|\vec{PQ} \times \vec{PR}| = 6\sqrt{2} \leftarrow$ area of parallelogram
    $\Rightarrow$ Area $\triangle = 3\sqrt{2}$

3. The length of the curve $r(t) = \langle \frac{2}{3}t^{\frac{3}{2}}, \cos 2t, \sin 2t \rangle$ on the interval $0 \le t \le 1$ is given by

    (a) 5

    (b) $\frac{2}{3}(5\sqrt{5} - 8)$ *(selected)*

    (c) $\frac{1}{3}\sqrt{13}$

    (d) $\frac{7}{3}$

    (e) Not enough information or none of the above

    Handwritten:
    $r'(t) = \langle t^{1/2}, -2\sin 2t, 2\cos 2t \rangle$
    $|r'(t)| = \sqrt{t + 4} \Rightarrow L = \int_0^1 \sqrt{t+4}\, dt = \frac{2}{3}(t+4)^{3/2}\Big|_0^1$
    $= \frac{2}{3}\left(5^{3/2} - 4^{3/2}\right)$
    $= \frac{2}{3}\left(5\sqrt{5} - 8\right)$ //

4. Let $\mathbf{v} = 3\mathbf{i} + 2\mathbf{j}$ and $\mathbf{w} = 5\mathbf{j} + \mathbf{k}$. The vector projection of $\mathbf{w}$ onto $\mathbf{v}$ is

    (a) $\frac{10}{13}\langle 3, 2, 0 \rangle$ *(selected)*

    (b) $\frac{17}{13}\langle 3, 2 \rangle$

    (c) $\frac{10}{13}\langle 0, 5, 1 \rangle$

    (d) $\frac{5}{13}\langle 0, 5, 1 \rangle$

    (e) Not enough information or none of the above

    Handwritten:
    $\text{Proj}_v w = \frac{\vec{w} \cdot \vec{v}}{|\vec{v}|^2}\vec{v} \Rightarrow \vec{w} \cdot \vec{v} = 0 + 10 + 0 = 10$
    $\vec{v} \cdot \vec{v} = 9 + 4 = 13$
    $\text{Proj}_v w = \frac{10}{13}\langle 3, 2, 0 \rangle$

5. A force is given by a vector $\mathbf{F} = 3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$ and moves a particle from the point $P(2, 1, 0)$ to the point $Q(4, 6, 2)$. If the unit of length is meters and the magnitude of force is measured in newtons, find the work done.

    (a) 6 joules

    (b) 36 joules *(selected)*

    (c) $\sqrt{50}$ joules

    (d) $\sqrt{1450}$ joules

    (e) Not enough information or none of the above

    Handwritten:
    $\vec{D} = \vec{PQ} = \langle 2, 5, 2 \rangle$
    $\vec{F} \cdot \vec{D} = 6 + 20 + 10 = 36$

2

1. True/False. Must CIRCLE the correct answer to receive credit.

(a) Any two non-parallel lines in $\mathbb{R}^3$ intersect.
- true
- **false**

(b) The line $\mathbf{r}(t) =< 3,-1,4 > +t < 6,-2,8 >$ passes through the origin.
- **true**
- false

$3+t6=0 \Rightarrow t=\frac{1}{2}$
$-1-t2=0 \Rightarrow t=\frac{1}{2}$ Yes, $\vec{r}(\frac{1}{2})=< 0,0,0>$
$4+t8=0 \Rightarrow t=\frac{1}{2}$

(c) The curve $\mathbf{r}(t) =< e^{-t}, \sin t, -\cos t >$ approaches a circle as $t \to \infty$.
- **true**
- false

Yes

(d) If $\mathbf{r}(t) = e^{-t^2} < 1,1,1 >$, then $\lim_{t\to\infty}\mathbf{r}(t) = \lim_{t\to-\infty}\mathbf{r}(t)$.
- **true**
- false

Yes, because $\lim_{t\to\pm\infty} e^{-t^2} = 0$

1. (24 points) An athlete throws a ball at an angle of $\frac{\pi}{4}$ to the horizontal at an initial speed of $43\frac{ft}{sec}$. It leaves his hand $7ft$ above the ground. Assume $g = 32ft/sec^2$.

$\vec{v}(0)= 43 < \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}> = <v_0, v_0>$
$\vec{r}(0) = < 0,7> = <x_0, y_0>$

(a) Where is the ball 2 seconds later?

$\vec{a} = < 0,-g>$
$\vec{v} = < v_0, -gt + v_0>$
$\vec{r} = < v_0 t + x_0, -\frac{gt^2}{2} + v_0 t + y_0>$

$\Rightarrow$

$\vec{r}(t) = < \frac{43\sqrt{2}}{2}t, -16t^2 + \frac{43\sqrt{2}}{2}t + 7>$
$\vec{r}(2) = < 43\sqrt{2}, -64 + 43\sqrt{2} + 7>$
$\boxed{\vec{r}(2) = < 43\sqrt{2}, 43\sqrt{2} - 57>}$
$60.811..., 3.8112...$

(b) How high does the ball go?

Maximum hight when $t^* = v_0/g$

$t^* = \frac{43\sqrt{2}}{64} s \Rightarrow y_{max} = -16\left(\frac{43\sqrt{2}}{64}\right)^2 + \frac{43\sqrt{2}}{2} \cdot \frac{43\sqrt{2}}{64} + 7$

$.9502..s$

$\boxed{y_{max} \cong 21.4453}$

(c) Where does the ball land?

The ball lands when $y(t)=0 \Rightarrow -\frac{gt^2}{2} + v_0 t + y_0 = 0$

$t^{**} = \frac{-v_0 \pm \sqrt{v_0^2 + 2y_0 g}}{-g} \cong 2.108 s$

$x_{max} = v_0 t^{**} + x_0 \cong \boxed{64.092 ft}$

3

2. (25 points) All following questions refer to a particle whose trajectory in time is given by $\mathbf{r}(t) = < 2\cos t, 2\sin t, t >$. The unit of length is meters and the unit of time is seconds.

(a) Find the speed at $t = \pi$.

[6 pts]

$\vec{r}'(t) = <-2\sin t, 2\cos t, 1>$

$|\vec{r}'(t)| = \sqrt{5}$ $\Rightarrow$ speed at $t = \pi$ is $\boxed{\sqrt{5}\, m/s}$

(b) Find the unit tangent vector at $t = \pi$.

[6 pts]

$\vec{T}(t) = \dfrac{\vec{r}'(t)}{|\vec{r}'(t)|} = \dfrac{1}{\sqrt{5}} <-2\sin t, 2\cos t, 1>$

at $t = \pi$ $\Rightarrow$ $\boxed{\vec{T}(\pi) = \dfrac{1}{\sqrt{5}} <0, -2, 1>}$

(c) What is the angle between the acceleration and velocity vectors at all times?

[6 pts]

$\vec{a}(t) = \vec{r}''(t) = <-2\cos t, -2\sin t, 0>$

$\vec{r}' \cdot \vec{r}'' = 0$ $\Rightarrow$ $\boxed{\text{angle is } \tfrac{\pi}{2} \text{ for all times}}$

(d) Find the tangent line to the trajectory at the point $(0, 2, \frac{\pi}{2})$.

[7 pts]

The particle passes by $(0, 2, \pi/2)$ when $t = \pi/2$

The tangent vector at $\pi/2$ is $\vec{r}'(\pi/2) = <-2, 0, 1>$

$\Rightarrow$ tangent line is $\boxed{<0, 2, \pi/2> + t <-2, 0, 1>}$

4