Practice Final

MAT 267	Student Name :
Instructor :	Student ID :
Final Exam 2014	Class Time :

Honor Statement

By signing below I confirm that I have neither given nor received any unauthorized assistance on this exam. This includes any use of a graphing calculator beyond those uses specifically authorized by the School of Mathematical and Statistical Sciences and my instructor. Furthermore, I agree not to discuss this exam with anyone until the exam testing period is over. In addition, my calculator's memory and menus may be checked at any time and cleared by any testing center proctor or School of Mathematical and Statistical Sciences instructor.

Signature

Instructions:

1. The exam consists of two parts: multiple choice, worth 60%, and free response (show your work), worth 40%. Please read each problem carefully.
2. There are 10 multiple choice questions worth 6 points each. Please fill in the table provided.
3. Provide complete and well-organized answers in the free response section.
4. Answers in the free response section without supporting work will be given zero credit. Partial credit is granted only if work is shown.
5. No calculators with Qwerty keyboards or ones like the Casio FX-2, TI-89, TI-92, or TI-*n*spire that do symbolic algebra may be used.
6. Proctors reserve the right to check calculators.
7. Please request scratch paper from me if you need it.
8. The use of cell phones is prohibited. TURN YOUR CELL PHONE OFF! Do not allow your cell phone to ring while you are taking the exam. Do not use the calculator on your cell phone. If a proctor sees you using a cell phone, they will take your exam and you will be reported to the Dean of Students for cheating.
9. *PLEASE NOTE:* "Any student who accesses a phone or any internet-capable device during an exam for any reason automatically receives a score of zero on the exam. All such devices must be turned off and put away and made inaccessible during the exam."

Read all directions carefully! Be neat, and box all answers. Points will be deducted for not following directions, sloppiness or lack of relevant work shown.

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Section I: Multiple Choice. Write your answers in the table provided. If you think the answer is "none of the above", write in E. (6 pts each)

1. Find the angle (in degrees) between the vectors $< -1,0,3 >$ and <3,2,7>. (Round to two decimal places.)

   u v

   $\cos^{-1}\left(\frac{u \cdot v}{|u||v|}\right) = \cos^{-1}\left(\frac{18}{\sqrt{10}\sqrt{62}}\right)$

   A. 81.41° **B.** 40.20° **C.** 72.02° **D.** 43.71° E. None of these

2. Which one of the following are parametric equations for the tangent line to the curve

   $t=0$ $x = 1 + \sin t, y = e^t, z = t + 2$

   at the point (1,1,2)?

   $\frac{dr}{dt} = \langle \cos t, e^t, 1 \rangle$

   $= \langle 1, 1, 1 \rangle$ at $t=0$

   A. $x = 1 + t, \quad y = 1 + t, \quad z = 2 + t$

   B. $x = 1 + 3t^2, \quad y = 1 + 2t, \quad z = -1 + t$

   C. $x = 1, \quad y = 1 + t, \quad z = 2 - t$

   D. $x = 1 + 3t, \quad y = 1 - 2t, \quad z = 1 + t$

   E. None of these

3. Find the projection of $\mathbf{v} = \langle 1,2 \rangle$ onto the vector $\boldsymbol{u} = < 3,4 >$. That is, find $\text{proj}_{\boldsymbol{u}}\boldsymbol{v}$.

   A. $\langle \frac{33}{5}, \frac{44}{5} \rangle$ **B.** $\langle \frac{11}{25}, \frac{22}{25} \rangle$ **C.** $\langle \frac{3}{25}, \frac{4}{25} \rangle$ **D.** $\langle \frac{33}{25}, \frac{44}{25} \rangle$ E. None of these

$\text{proj}_u v = \left(\frac{u \cdot v}{u \cdot u}\right) u = \frac{11}{25} \langle 3, 4 \rangle = \left\langle \frac{33}{25}, \frac{44}{25} \right.$

4. Suppose $z = x^2 + y^3$ $x = e^{st}$, and $y = \ln(2s + 5t)$. Find $\frac{\partial z}{\partial s}$
(Do not simplify.)

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$$

$$= 2x\, t e^{st} + 3y^2 \frac{2}{2s+5t}$$

A. $2xe^{st}s + 3y^2 \frac{1}{2s+5t}$
B. $2xe^{st}t + 3y^2 \frac{1}{2s+5t}$
C. $2xe^{st}t + 3y^2 \frac{2}{2s+5t}$
D. $x^2e^{st}t + y^3 \frac{5}{2s+5t}$
E. None of these

5 Find the directional derivative of the function $f(x, y) = x^2 \sin y$ at the point $(1, \pi)$ in the direction of the vector $v = 5i + 12j$.

$\nabla f = \langle 2x\sin y, x^2\cos y\rangle = \langle 0, -1\rangle$

$u = \frac{5}{13}\vec{i} + \frac{12}{13}\vec{j}$

$\nabla f \cdot \vec{u}$

A. $-\frac{12}{13}$ **B.** $\frac{17}{13}$ **C.** $-\frac{5}{13}$ **D.** 1 **E.** None of these

6. Use Green's theorem to evaluate $\int_C (xe^{x^2})dx + (x + \ln y)dy$, where C is the circle $x^2 + y^2 = 1$ oriented in positive direction.

$$\iint_D 1\, dA = \pi \cdot 1^2$$

A. 0 **B.** π **C.** 2π **D.** 1 **E.** None of these

7. Let $F(x, y, z) = < 5x, 2xy, 7z >$. Find the divergence of and curl of F.

$\text{div } F = 5 + 2x + 7 = 12 + 2x$

$$\text{curl } \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 5x & 2xy & 7z \end{vmatrix}$$

$$= 0\vec{i} + 0\vec{j} + 2y\vec{k}$$

A. div (F) = 12+2x and curl(F)= $0i + 0j + 2yk$

B. div (F) = 12+2x and curl(F)= $5i + 2yj + 7k$

C. div (F) = 14+2x and curl(F)= $0i + 2yj + 0k$

D. div (F) = 14 and curl(F)= $5i + 2yj + 7k$

E. None of these

8. For the surface with the parametric equations $r(s, t) = < 3t, st^3, s + t >$ find an equation of the tangent plane to the surface at (6,8,3) $t = 2$ $s = 1$

$r_s = \langle 0, t^3, 1\rangle$ at $\langle 0, 8, 1\rangle$

$r_t = \langle 3, 3st^2, 1\rangle = \langle 3, 12, 1\rangle$

$$r_s \times r_t = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 8 & 1 \\ 3 & 12 & 1 \end{vmatrix}$$

A. $-18(x - 6) + 324(y - 8) - 11(z - 3) = 0$

B. $-4(x + 6) + 3(y + 8) - 24(z + 3) = 0$

C. $-4(x - 6) + 3(y - 8) - 24(z - 3) = 0$

D. $-18(x + 6) + 324(y + 8) - 11(z + 3) = 0$

E. None of these

$$= -4\vec{i} + 3\vec{j} - 24\vec{k}$$

$v(t) = \langle t + C_1, \frac{1}{2}t^2 + C_2, C_3 \rangle$ $\quad v(0) = \langle C_1, C_2, C_3 \rangle$

9. Find the velocity vector of a particle with acceleration $\boldsymbol{a}(t) = \langle 1, t, 0 \rangle$ and initial velocity $\boldsymbol{v}(0) = \langle 1,2,3 \rangle$

A. $\langle t + 1, \frac{1}{2}t^2 + 2, 3 \rangle$ **B.** $\langle -1, 3, 3 \rangle$ **C.** $\langle t, \frac{1}{2}t^2, 3 \rangle$

D. $\langle t + 1, \frac{1}{2}t^2 + 2, 5 \rangle$ **E.** None of these.

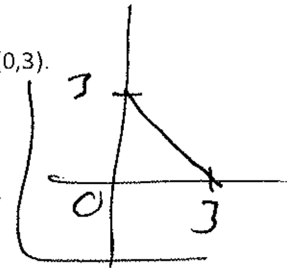
$\int_0^3 \int_0^{3-x} 2y \, dy \, dx$

10. Evaluate the double integral $\iint_D 2y \, dA$ where D is the triangular region with vertices (0,0), (3,0), (0,3).

$y^2 \Big|_0^{3-x} = x^2 - 6x + 9$

A 9 **B.** 18 **C.** −18 **D.** −9 **E.** None of these

$\int x^2 - 6x + 9 \, dx = \left(\frac{1}{3}x^3 - 3x^2 + 9x\right)\Big|_0^3 = 9 - 27 + 27$



PART II – Free Response. You must show all work for credit. Box your final answers.

11. [10 pts] Find the local maximum and minimum values and saddle point(s), if any of the function
$$f(x, y) = x^2 + 3y^2 + 2xy - 8x - 12y$$

SYSTEM

$f_x = 2x + 2y - 8 = 0$

$f_y = 6y + 2x - 12 = 0$

$f_{xx} = 2$ $\quad f_{xy} = 2$

$f_{yy} = 6$

$D = 2 \cdot 6 - 2^2 =$

$x + y = 8$

$2x + 6y = 12$

$x + y = 8$

$x + 3y = 6$

$-2y = 2$

$y = -1$

$x = 9$

Critical pt	D = 8	f_{xx}	Type
(9, −1)	+	+	Min Value

Answer: There is a minimum at the pt (9, −1)

12. [10 pts] Let $\boldsymbol{F}(x, y, z) = (yz)\boldsymbol{i} + \left(xz + \frac{1}{y}\right)\boldsymbol{j} + (xy + 3z^2)\boldsymbol{k}$

a) Find a potential function for $\vec{F}$.

$f_x = yz \Rightarrow f = xyz + g(y,z) \Rightarrow f_y = xz + g_y(y,z)$

$f_y = xz + \frac{1}{y} \rightarrow g_y(y,z) = \frac{1}{y}$

so $g(y,z) = \ln|y| + h(z)$

$f_z = xy + 3z^2$

so $f = xyz + \ln|y| + h(z)$

$f_z = xy + h'(z)$

$h'(z) = 3z^2$

$h(z) = z^3 + C$

ANSWER

$f = xyz + \ln|y| + z^3 + C$

b.) Use your answer to part (a) above to find $\int_C \vec{F} \cdot d\vec{r}$, where C is a curve from (2, 1, 3) to (1, 1, 5)

$$\int_C \vec{F} \cdot d\vec{r} = f(1,1,5) - f(2,1,3)$$

$$= (5 + 125) - (6 + 27)$$

$$= 130 - 33 = 97$$

13. [10 pts] Find the surface area of the part of the plane $2x - 3y + z = 100$ that lies inside of the cylinder $x^2 + y^2 = 16$.

$$z = 100 - 2x + 3y$$

$$A = \iint_D \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dA$$

$$= \iint_D \sqrt{1 + 4 + 9}\, dA = \sqrt{14} \iint_D 1\, dA$$

$$= (\sqrt{14})(\text{Area of the circle})$$

$$= \boxed{16\pi\sqrt{14}}$$

14. [10 pts] Find the flux of the vector field $E = \langle -y, x, 2z \rangle$ outward through the sphere $x^2 + y^2 + z^2 = 1$.

$$r(\varphi,\theta) = \langle \sin\varphi\cos\theta,\ \sin\varphi\sin\theta,\ \cos\varphi \rangle$$

$$r_\varphi \times r_\theta = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos\varphi\cos\theta & \cos\varphi\sin\theta & -\sin\varphi \\ -\sin\varphi\sin\theta & \sin\varphi\cos\theta & 0 \end{vmatrix}$$

$$= \langle \sin^2\varphi\cos\theta,\ \sin^2\varphi\sin\theta,\ \sin\varphi\cos\varphi(\cos^2\theta + \sin^2\theta) \rangle$$

$$= \langle \sin^2\varphi\cos\theta,\ \sin^2\varphi\sin\theta,\ \sin\varphi\cos\varphi \rangle$$

$$E(r(\varphi,\theta)) = \langle -\sin\varphi\sin\theta,\ \sin\varphi\cos\theta,\ 2\cos\varphi \rangle$$

$$\iint \vec{E}\cdot dS = \int_0^{2\pi}\int_0^{\pi} -\sin^3\varphi\,\cancel{\sin\theta\cos\theta} + \sin^3\varphi\,\cancel{\sin\theta\cos\theta} + 2\cos^2\varphi\,(\sin\varphi)\,d\varphi\,d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi} 2\cos^2\varphi\sin\varphi\,d\varphi\,d\theta$$

$$= \int_0^{2\pi} \frac{4}{3}\,d\theta = \frac{4}{3}\theta\Big|_0^{2\pi}$$

$$= \frac{8\pi}{3}$$

$$\int_0^{\pi} 2\cos^2\varphi\sin\varphi\,d\varphi,\quad u = \cos\varphi$$

$$= \int_1^{-1} 2u^2(-du)$$

$$= \left(-\frac{2}{3}u^3\right)\Big|_1^{-1}$$

$$= -\frac{2}{3}\cos^3\varphi\Big|_0^{\pi}$$

$$= \frac{4}{3}$$