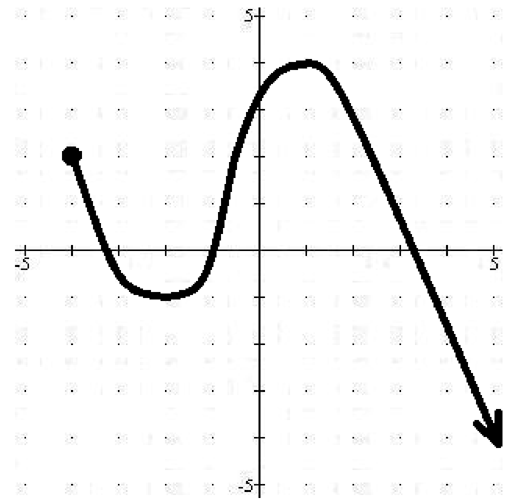
MAT 251 – Exam 2 Review

1) Find all relative extrema of the following function.
Answer in exact values. NO DECIMAL APPROXIMATIONS.

$$f(x) = -4x^3 - 3x^2 + 36x + 12$$



(Problems 2 – 4) Use the graph of f(x) to the right to answer the following.
True or False problems.

2) f(x) achieves a relative maximum when x = -4.
3) The absolute maximum value of f(x) is 4.
4) The absolute minimum value of f(x) is $-\infty$.

5) Given the function $f(x) = \dfrac{2x^3}{3} - x^2 + \dfrac{x}{2}$

A) List all points at which any extrema occur and classify them.
B) List all points of inflection.

6) Given the function $f(x) = \dfrac{x^2 + 4}{5x}$

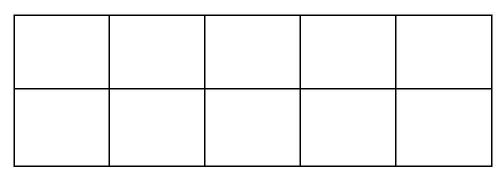
A) Find the absolute maximum value on the interval [1, 5].
B) Find the absolute minimum value on the interval [1, 5].

7) Maximize R = $3x^2 - y^2$ subject to 2x + y = 32, and find the corresponding values of x and y.

8) Square pieces of cardboard will be cut out of the corners of a 32 inch by 40 inch piece of cardboard, then the sides will be folded up to create an open-top box. Find the maximum volume of the box. Round to 2 decimal places. (Find the dimensions also)

9) A box will have a square base and no top. The material used to construct the base costs $.50 per square foot, while the material for the sides costs $.30 per square foot. If the box must have a volume of 18 cubic feet, then find the minimum cost and the dimensions that minimize the cost.

10) If we are fencing a garden into 10 sections (as shown to the right) and we are limited to 200 meters of fencing, then maximize the area of the garden.



11) We are fencing a garden into 10 sections (as shown to the right). The vertical fencing (as shown in the picture) costs $8.50 per meter, and the horizontal costs $12.00 per meter. The area must be 400 m^2 Minimize the cost of the fencing and find the dimensions.

Use implicit differentiation to find $\dfrac{dy}{dx}$

12) $4x^3 + y^2 - 7 = 8xy - 3x + 2y$ 13) $x\cos y - 4y^3 = 9x + 5$

14) $\dfrac{3 - y^2}{4x + y} = 2x - 1$ 15) $xe^{3y} + \ln y = 5x - 8y$

16) The volume of a spherical balloon is given by $V = \dfrac{4}{3}\pi r^3$. The radius is decreasing at a rate of 1.2 inches per second. How fast is the volume decreasing when the radius is 4 inches? (2 decimal places)

17) The length of a rectangle is increasing at a rate of 3 inches per second, but the area is decreasing at a constant 8 in^2 per second. What is happening to the width when the area is 200 in^2 and the length is 25 inches?

18) Starting at a point 15 feet directly south of Sue, Jacob starts jogging due east at a rate of 4 feet per second. How fast is the distance between them increasing after 4 seconds?

19) The surface area of a cylinder is given by $A = 2\pi r^2 + 2\pi rh$. If the area is increasing at a constant 50 cm^2 per minute, while the radius is increasing at 3 cm per minute, then what is happening to the height when the height is 20 cm and the radius is 4 cm?

20) Assuming that x, y, and z are positive, use the properties of logarithms to expand $\log_3 \sqrt{\dfrac{x^4(y+3)}{81z^7}}$ as much as possible. Simplify as much as possible.

21) Write $5\ln x - 8\ln y + \dfrac{1}{3}\ln(z+4)$ as a single logarithm.

22) Given $f(x) = \left(5e^{6x^2} + 4\right)^6$, find $f'(x)$

23) Given $f(x) = \ln(5e^{4x} + 3)$, find $f'(x)$

24) Given $f(x) = \dfrac{e^{3x}}{\sin x}$, find $f'(x)$

25) Given $f(x) = \sec(3\ln x)$, find $f'(x)$

26) Solve for t (Exact value, no decimals). $5e^{8t} = 14700$

27) The average walking speed of a person living in a city with population p (in thousands) is given by $v(p) = .37\ln(p) + .86$ Where v is in feet per second. Use this info to answer the next 2 questions.
A) Find the average walking speed of people living in a city with population 70,000.
B) If the average walking speed in a city is 3.4 feet per second, then estimate the population of the city. Round your estimate to the nearest thousand.

28) The population of bunnies on Grassygreen Island (B) experiences exponential growth with a relative growth rate of 1.4% per day. Right now, the population is 140. Use the model $P(t) = P_o e^{kt}$
A) Find a formula for the population t days from now. (Find the growth model.)
B) What will the population be after 30 days? (Round to the nearest bunny.)
C) How long will it be until the population reaches 4000? Round to the nearest day.
D) How fast is the population growing after exactly 45 days?

29) A mummy was recently found in Mesa. It now contains 73.8% of its original Carbon-14 content. About how old is the mummy? Round to the nearest year. Assume that Carbon-14 has a half-life of about 5730 years.

30) How old is a skeleton that has lost 97% of its carbon-14? Assume that Carbon-14 has a half-life of about 5730 years.

31) Element X decays exponentially. 20 years ago, a sample with 65 g of element X now only has 43 g. How much will be left 50 years from now?

32) A 210° cup of coffee is placed on a table in a climate-controlled room with the temperature set at a constant 73°. After 6 minutes, the temperature of the coffee had dropped to 150°. Find the temperature of the coffee exactly 21 minutes after it is placed on the table.

Find the derivative of each of the following.

33) $f(x)=3^{5x}$ 34) $f(x)=\log_2(4x-5)$ 35) $f(x)=x(10^x)$ 36) $\log(x^2+1)$

37) $f(x)=\dfrac{\log_5 x}{x^2}$ 38) $f(x)=\log_3 3^x$

39) Find the absolute maximum and minimum values of f(x) = $4x^3 - 8x^2 + 1$ over the interval [-1, 1]

Solutions

1) f(x) has a relative maximum at the point $\left(\dfrac{3}{2},\dfrac{183}{4}\right)$ and a relative minimum at the point $(-2,-40)$

2) False 3) True 4) False 5A) There are no maxima or minima (neither relative nor absolute)

B) Point of Inflection at $\left(\dfrac{1}{2},\dfrac{1}{12}\right)$ 6A) $\dfrac{29}{25}$ (when x = 5) B) $\dfrac{4}{5}$ (when x = 2)

7) The maximum value is 3072 when x = 64 and y = -96

8) The maximum volume is about 3360.88 in^2 when the base is 28.22 by 20.22 inches and the height is 5.89 inches

9) The minimum cost is about $11.63 when the base is 2.78 ft by 2.78 ft and the height is 2.32 ft.

10) The maximum area is about 555.56 m^2 when the dimensions are 16.67 m (vert) by 33.33 m (horiz)

11) The minimum cost is about $1713.94 when the dimensions are 16.80 m (vert) by 23.80 m (horiz)

12) $\dfrac{12x^2-8y+3}{8x-2y+2}$ 13) $\dfrac{\cos y-9}{x\sin y+12y^2}$ 14) $\dfrac{4-16x-2y}{2x+2y-1}$ 15) $\dfrac{5y-ye^{3y}}{3xye^{3y}+8y+1}$

16) The volume is decreasing at a rate of about 241.27 in^3 per second.

17) The width is decreasing at a rate of 1.28 inches per second.

18) about 2.92 feet per second 19) The height is decreasing at a rate of about 19.01 cm per second.

20) $2\log_3 x+\dfrac{1}{2}\log_3(y+3)-\dfrac{7}{2}\log_3 z-2$ 21) $\ln\left(\dfrac{x^5\sqrt[3]{z+4}}{y^8}\right)$

22) $360xe^{6x^2}\left(5e^{6x^2}+4\right)^5$ 23) $\dfrac{20e^{4x}}{5e^{4x}+3}$ 24) $\dfrac{e^{3x}(3\sin x-\cos x)}{\sin^2 x}$ 25) $\dfrac{3\sec(3\ln x)\tan(3\ln x)}{x}$

26) $t=\dfrac{\ln(2940)}{8}$ 27A) 2.43 feet per second B) 958000

28A) $P(t) = 140e^{.014t}$ B) 213 C) 239 days D) at about 3.68 bunnies per day

29) about 2512 years old 30) about 28987 years old 31) about 15.31 g 32) about 92°

33) $5(3^{5x})\ln 3$ 34) $\dfrac{4}{(4x-5)\ln 2}$ 35) $10^x(x\ln 10 + 1)$ 36) $\dfrac{2x}{(x^2+1)\ln 10}$

37) $\dfrac{1-2(\log_5 x)(\ln 5)}{x^3\ln 5}$ 38) 1

39) The absolute maximum is 1 (when x = 0)

The absolute minimum is -11 (when x = -1)