MAT 211 Exam 2 Review Questions with Answers

Section 5.1/5.2 Matrices and Matrix Multiplication

1. Let A=$\begin{bmatrix} -2 & 0 \\ -1 & 2 \\ -1 & -1 \end{bmatrix}$; B=$\begin{bmatrix} -1 & 1 & 1 \\ -1 & 0 & x \end{bmatrix}$; C=$\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$; I=$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$; E=$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

   Perform the following calculations. Simplify terms where possible. If a calculation is not possible, state "Not Possible".

   a. $2A^T+3B$ b. $-2A+3B$
   c. AB d. BA
   e. $2B^TC-4A$ f. B^2

2. Given that A= $\begin{bmatrix} 1 & 2 \\ 3 & x \end{bmatrix}$; C = $\begin{bmatrix} 1 & 0 \\ 0 & y \end{bmatrix}$; D= $\begin{bmatrix} 1 & 4 \\ 9 & x^2 \end{bmatrix}$; F = $\begin{bmatrix} 1 & 0 \\ 0 & y^2 \end{bmatrix}$

   Answer **True** or **False** to the following:

   a. $A^2 = D$ ________________ b. $C^2 = F$ _______________.

3. Given that the dimensions of matrix A is 5 x 4, B is 4 x 4, C is 3 x 4 and D is 4 x 5, Find the dimensions of the following. (If the calculation is not possible, enter NOT POSSIBLE).

   d. $(A-C^T)C$ e. $-2DA+4B$ f. $3D^TB-A$

Section 5.1/5.2 Gauss-Jordan Row Reduction

4. Use Gauss-Jordan row reduction to solve the given system of equations. Express your answer as an ordered pair (x, y). If there is no solution, enter NO SOLUTION. If the system is dependent, express your answer in terms of *x*, where *y* = *y*(*x*) and where *z* = *z*(*x*). Make sure to show all your work.

   a. $\begin{Bmatrix} 2x+3y=1 \\ 5x+4y=6 \end{Bmatrix}$ b. $\begin{Bmatrix} 5x+y=6 \\ x-y=-2 \\ 3x+3y=10 \end{Bmatrix}$ c. $\begin{Bmatrix} x-y+2z=9 \\ x-y+3z=4 \end{Bmatrix}$

5. The following matrices represent systems of 2 equations with 2 variables. Gauss-Jordan elimination was used to arrive at the given matrices. Express the solution indicated by each matrix in the form (x, y) or state that no solution exists. If the system is dependent, express your answer as an ordered pair in terms of x where y=y(x).

   a. $\begin{bmatrix} 1 & -3 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$.

6. The following matrices represent systems of 3 equations with 3 variables. Gauss-Jordan elimination was used to arrive at the given matrices. Express the solution indicated by each matrix in the form (x, y, z) or state that no solution exists. If the system is dependent, express your answer as an ordered pair in terms *x*, where *y* = *y*(*x*) and where *z* = *z*(*x*).

$a.\ \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$ b. $\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$

Section 5.3/3.6 Matrix Inversion and Determinants

7. Evaluate each determinant

a. $\begin{bmatrix} -1 & 2 \\ -1 & -2 \end{bmatrix}$ b. $\begin{bmatrix} 5 & -1 \\ 0 & 0 \end{bmatrix}$ c. $\begin{bmatrix} -2 & 5 & -2 \\ -1 & 1 & 0 \\ -3 & -2 & 2 \end{bmatrix}$

8. For each matrix in problem 7, state if an inverse exists.

9. Find the inverse of each matrix in problem 7, if it exists.

10. For what value(s) of x does the matrix have an inverse?

a. M= $\begin{bmatrix} 5 & -2 \\ 10 & -x \end{bmatrix}$ b. M= $\begin{bmatrix} 2 & x \\ 4 & x+1 \end{bmatrix}$

11. Solve the given system of equation using *Matrix Inversion method***.**

a. $\begin{aligned} 3x + 6y &= 3 \\ 2x - y &= 7 \end{aligned}$ b. $\begin{aligned} 3a + 3b &= 180 \\ 6a + 30b &= 600 \end{aligned}$ c. $\begin{aligned} -x + y &= -1 \\ y - z &= 6 \\ x + z &= -1 \end{aligned}$

12. Given the matrix equation: $CX = D$, which of the following represents an appropriate calculation to find the solution for X?
a. $X = D^{-1}C$ b. $X = DC^{-1}$ c. $X = C^{-1}D$ d. $X = CD^{-1}$

Section 3.7 Cramer's Rule

13. Solve the given system of linear equations in questions 11 using *Cramer's Rule*.

14. Solve the given problem using *Cramer's Rule*.
You manage an ice cream factory that makes two flavors: Creamy Vanilla and Continental Mocha. Into each quart of Creamy Vanilla go 2 eggs and 3 cups of cream. Into each quart of Continental Mocha go 1 egg and 3 cups of cream. You have in stock 550 eggs and 1050 cups of cream. How many quarts of each flavor should you make in order to use up all the eggs and cream?

Section 7.1/7.2 Sets and Cardinality

15. Let S = {Arthur, Dirk, Frans, Henrika, Johan, Klaas, Sarie, Tina} be the universal set with subsets
    A = {Dirk, Frans, Johan, Sarie},
    B = {Frans, Henrika, Klaas, Sarie, Tina}, and
    C = {Frans, Tina}.

    a. What is $n(\mathrm{A} \cup C)$?
    b. What is $n(A \cap B')$?
    c. What is $n[(A \cap B)']$?

16. If $n(A) = 50$, $n(B) = 40$, and $n(A \cup B) = 80$, Find $n(A \cap B)$?

Section 7.3/7.4 Counting Methods: Decision Algorithm, Permutation and Combinations

17. The local diner offers a meal combination consisting of an appetizer, a soup, a main course, and a dessert. There are 5 appetizers, 5 soups, 6 main courses, and 6 desserts. Your diet restricts you to choosing between a dessert and an appetizer. (You cannot have both.) Given this restriction, how many three-course meals are possible?

18. How many different sequences can be formed that use all the letters of the given words?

    a. CHOSEN b. ELEPHANT c. EXCELLENCE

19. A 6-digit password is to be formed from the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. How many different passwords are possible
    a. if the digits are repeated?
    b. if there are no repeated digits?

20. In a lottery you must select 6 out of 50 numbers, how many ways are there to do this.

21. A professor wants to award prizes for 1st, 2nd, 3rd, 4th and 5th in the class of 30. In how many ways can the prizes be awarded (assume no two students tie)?

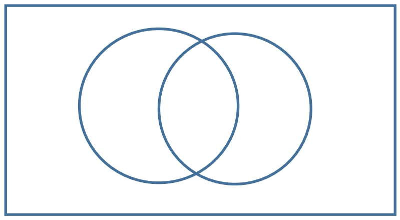
Section 8.1/8.2/8.3 Basic Probability

22. Consider the experiment where a fair coin is flipped 3 times. Let A represent the event that Heads come up ***at most 2*** in the three flips. Find $P(A)$ (the probability of A).

23. Consider the experiment where two fair, distinguishable dice are rolled.
    Let B represent the event that the sum of the two numbers is 5. Find $P(B)$ (the probability of B).

24. Use the given information to find the indicated probability.
$P(A \cup B) = 0.6$, $P(A) = 0.3$, $P(A \cap B) = 0.2$, Find $P(B)$?

25. According to the weather service, there is a 75% chance of rain in New York and a 30% chance of rain in Honolulu. There is also a 10% chance that it will rain in both cities.
    a) Complete the Venn diagram by filling in all four regions with the appropriate percentage values. Make sure that all four regions add up to 100%.



    b) What is the probability that it will rain in *neither* of the two cities? Express your answer as an exact decimal.
    c) What is the probability that it will rain in *exactly one* of the two cities? Express your answer as an exact decimal.

26. A survey conducted by the Bureau of Labor Statistics found that 67% of high school graduating class of 2007 went on to college the following year, while 44% of the class was working. Furthermore, 92% were either in college or working, or both.
    a. What percentage went on to college and work at the same time?
    b. What percentage went on to college but not work?
    c. What percentage were neither in college nor work?

Section 8.4 More Probability

27. A bag contains 5 red, 6 green and 4 blue marbles. Four are drawn out at random What is the probabilities of the following events expressing each as a fraction in the lowest term.

(a) all are green (b) all are the same color (c) at least 3 are red

(d) two blue and one of each of the other colors (e) NOT all are green

28. Suzan sees a bag of marbles; she grabs a handful at random. She has seen a bag containing 4 red marbles, 3 green ones, 2 white ones, and 1 purple one. She grabs 5 of them. Find the probability of the following event, expressing it as a fraction in lowest terms.
(a) She has all the red ones (b) She has at least one white

(c) two red and one of each of the other colors (d). She does not have all the red ones.

Answers

1. a. $2A^T+3B=\begin{bmatrix} -7 & 1 & 1 \\ -3 & 4 & -2+3x \end{bmatrix}$ b. -2A+3B is not possible

c. $AB = \begin{bmatrix} 2 & -2 & -2 \\ -1 & -1 & -1+2x \\ 2 & -1 & -1-x \end{bmatrix}$ d. $BA= \begin{bmatrix} 0 & 1 \\ 2-x & -x \end{bmatrix}$

e. $2B^TC-4A = \begin{bmatrix} 6 & 0 \\ 4 & -6 \\ 2x+4 & 6-2x \end{bmatrix}$ f. B^2 is not possible

$2a.$ $A^2 = D$ __False__ 2b. $C^2 = F$ __True .__

3a. Not possible 3b. 4x4 3c. 5x4

4a. (2, -1). 4b. $(\frac{2}{3}, \frac{8}{3})$ 4c. $(x, x-19, -5)$

5a. No solution 5b. (x, 2- x).

6a. (x, y, z) = (3,2,0) 6b. No solution

7a. Determinate =4 7b. Determinate =0 7c. Determinate = -4

8a. YES 8b. NO 8c. YES

9a. $\begin{bmatrix} \frac{-1}{2} & \frac{-1}{2} \\ \frac{1}{4} & \frac{-1}{4} \end{bmatrix}$ 9b. No inverse 9c. $\begin{bmatrix} \frac{-1}{2} & \frac{3}{2} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{5}{2} & \frac{-1}{2} \\ \frac{-5}{4} & \frac{19}{4} & -\frac{3}{4} \end{bmatrix}$

10a. All values except 4 10b. All values except 1.

11a. (x, y) = (3, -1) 11b. (a, b) = (50,10) 11c. (x, y, z) = (3,2, -4)

12. c. $X = C^{-1}D$

13a. (x, y) = (3, -1) 13b. (x, y) = (50,10) 13c. (x, y, z) = (3,2, -4)

14. 200 quarts of Creamy Vanilla and 150 quarts of Continental Mocha

15a. $n(A \cup C)= 5$ 15b. $n(A \cap B')=2$ 15c. $n[(A \cap B)']=6$

16. $n\ (A \cap B)=10$ 17. 330

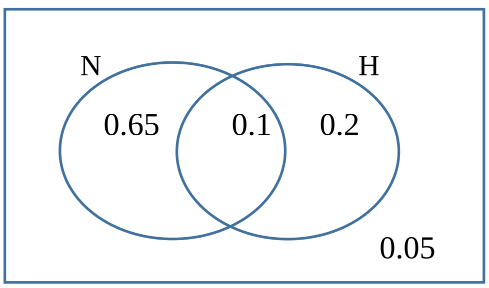
18a. 720 18b. 20160 18c. 37800

19a. 10^6 19b. 151200

20. 15,890,700 21. 17,100,720

22. $\frac{7}{8}$ 23. $\frac{1}{9}$ 24. 0.5

25a. N= New York H= Honolulu



25b. 5% 25c. 85%

26a. 19% 26b. 48% 26c. 8%

27a. $\frac{1}{91}$ 27b. $\frac{1}{65}$ 27c. $\frac{1}{13}$ 27d. $\frac{12}{91}$ 27e $\frac{90}{91}$

28a. $\frac{1}{42}$ 28b. $\frac{7}{9}$ 28c. $\frac{1}{7}$ 28d. $\frac{41}{42}$