MAT266 Spring 2019 Exam 3 Review

1. Suppose a series $\sum_{n=1}^{\infty} a_n$ meets the conditions to use the ratio test. What, if any, is the most specific conclusion that may be drawn if:
   (a) $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \frac{5}{3}$
   (b) $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{2}$
   (c) $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1$
   (d) $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 0$
2. Consider the series $\sum_{n=6}^{\infty} \frac{n+9}{n!}$. Find $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$.
3. Consider the series $\sum_{n=6}^{\infty} \frac{(5n+3)5^{n+2}}{8^n}$. Find $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$.
4. Determine whether the series $\sum_{n=0}^{\infty} \frac{(-5)^n}{(n+1)3^{2n+1}}$ is absolutely convergent, conditionally convergent, or divergent.
5. Determine whether the series $\sum_{n=0}^{\infty} \frac{n\cdot 5^n}{3^{2n+1}}$ is absolutely convergent, conditionally convergent, or divergent.
6. Determine whether the series $\sum_{n=1}^{\infty} \frac{e^{n-6}}{\sqrt{n+7}(n+8)!}$ is absolutely convergent, conditionally convergent, or divergent.
7. Find the largest **open** interval of convergence and the radius of convergence for $\sum_{n=1}^{\infty} \frac{(3x)^n}{n!}$.
8. Find the largest **open** interval of convergence and the radius of convergence for $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n+7}$.
9. Find the largest **open** interval of convergence and the radius of convergence for $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n\cdot 2^n}$.
10. Find the largest **open** interval of convergence and the radius of convergence for $\sum_{n=1}^{\infty} \frac{(5x)^n}{n^4}$.
11. Find the largest **open** interval of convergence and the radius of convergence for $\sum_{n=1}^{\infty} \frac{(-1)^n x^{n+1}}{3^{n-1}}$.

12. Find the largest **open** interval of convergence and the radius of convergence for $\sum_{n=1}^{\infty} n!(3x+1)^n$.

13. Find the power series representation of the function $f(x) = \frac{3}{4-x}$ for $|x| < 4$ by using the fact that $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ for $|x| < 1$.

14. Find the power series representation of the function $f(x) = \frac{x^2}{2+x}$ for $|x| < 2$ by using the fact that $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ for $|x| < 1$.

15. Find the power series representation of the function $f(x) = \frac{x^3}{4-x}$ for $|x| < 4$ by using the fact that $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ for $|x| < 1$.

16. Find the power series representation of the function $f(x) = \frac{x}{\frac{1}{2}-x}$ for $|x| < \frac{1}{2}$ by using the fact that $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ for $|x| < 1$.

17. Find a power series representation for $f(x) = \ln(1+x)$. *Hint:* $\ln(1+x) = \int \frac{1}{1+x}\, dx$.

18. Find a power series representation for $f(x) = \ln(1-x)$. *Hint:* $\ln(1-x) = \int \frac{-1}{1-x}\, dx$.

19. Find a power series representation for $f(x) = \arctan x$. *Hint:* $\arctan x = \int \frac{1}{1+x^2}\, dx$.

20. Find the first three non-zero terms of the Taylor series for $f(x) = x^{-3}$ about $a = 1$.

21. Find the first three non-zero terms of the Maclaurin series for $f(x) = (1+x)^{-2}$.

22. Find the first three non-zero terms of the Maclaurin series for $f(x) = (1-x)^{-2}$.

23. A function $f(x)$ has a Maclaurin Series on $(-1, 1)$. If possible, find the first three non-zero terms of the Maclaurin series if $f(0) = 5$; $f'(0) = -2$; $f''(0) = 2$.

24. A function $f(x)$ has a Maclaurin Series on $(-1, 1)$. If possible, find the first three non-zero terms of the Maclaurin series if $f(0) = 0$; $f'(0) = -1$; $f''(0) = 4$; $f'''(0) = 6$.

25. A function $f(x)$ has a Maclaurin Series on $(-1, 1)$. If possible, find the first three non-zero terms of the Maclaurin series if $f(0) = 2$; $f'(0) = 0$; $f''(0) = 1$.

26. Use the known Maclaurin series for $\sin x$ to find the first three **nonzero** terms of the Maclaurin series for $f(x) = \sin(x^2)$.

27. Use the known Maclaurin series for $\sin x$ to find the first three **nonzero** terms of the Maclaurin series for $f(x) = x\sin(3x)$.

28. Use the known Maclaurin series for $\cos x$ to find the Maclaurin series for $f(x) = \dfrac{x}{27}\cos(3x)$.

29. Use the known Maclaurin series for $\cos x$ to find the Maclaurin series for $f(x) = x\cos\left(\dfrac{x}{2}\right)$.

30. Use the known Maclaurin series for e^x to find the Maclaurin series for $f(x) = x^2e^{3x}$.

31. Use the known Maclaurin series for e^x to find the Maclaurin series for $f(x) = e^{-x^2}$.

32. The resistivity ρ of a given metal depends on temperature and can be modeled by the equation

$$\rho(t) = \rho_{20}e^{\alpha(t-20)}$$

where t is the temperature in $^\circ C$, and ρ_{20} and α are constants depending on the metal. At most temperatures, resistivity can be approximated by the second-degree (quadratic) Taylor polynomial centered at $t = 20$. Find an expression for the second-degree Taylor polynomial centered at $t = 20$.

33. Eliminate the parameter to find the Cartesian equation of the curve $x = e^t - 1$; $y = e^{2t}$.

34. Find the Cartesian equation of the curve $x = 5\cos t$; $y = 5\sin t$, $0 \le t \le 2\pi$.

35. Find the Cartesian equation of the curve $x = 5\sin t$; $y = 3\cos t$, $0 \le t \le 2\pi$.

36. Find the Cartesian equation of the curve $x = 2 + 3\cos t$; $y = 5 + 3\sin t$, $0 \le t \le 2\pi$.

37. Describe the parametric curve for the set of parametric equations $x = 5\cos t$; $y = 2\sin t$, $0 \le t \le 2\pi$. Include the starting point, the terminal point, and the direction.

38. Describe the parametric curve for the set of parametric equations $x = 2 + \cos t$; $y = 3 + \sin t$, $0 \le t \le \pi$. Include the starting point, the terminal point, and the direction.

39. For the parametric curve defined by $x = t^4 - t - 3$ and $y = 3\ln t - t^2 + 3$,
    (a) Algebraically find the equation of the line tangent to the curve at the point corresponding to $t = 1$.
    (b) Find all values of t where the tangent line to the graph of the curve is horizontal.

40. For the parametric curve defined by $x = 8t + \ln t$ and $y = 9t - \ln t$,
    (a) Algebraically find the equation of the line tangent to the curve at the point corresponding to $t = 1$.

(b) Find all values of t where the tangent line to the graph of the curve is horizontal.

41. If a projectile is fired with an initial velocity of v_0 at an angle of α above the horizontal and air resistance is assumed to be negligible, then its position after t seconds is given by the parametric equations

$$x = (v_0 \cos\alpha)t \qquad y = (v_0 \sin\alpha)t - \frac{1}{2}gt^2$$

where g is the acceleration due to gravity ($9.8\,\text{m/s}^2$), α is the angle of elevation, and v_0 is the initial velocity.

(a) Find dy/dx

(b) Suppose a projectile is fired with $v_0 = 200\,\text{m/s}$ at an angle $\alpha = 30°$. At what time does it reach its maximum height? *Round your answer to the nearest tenth of a second.*

42. Find the exact length of the curve defined by the set of parametric equations $x = \sin(5t)$, $y = \cos(5t)$ for $0 \le t \le \frac{\pi}{4}$.

43. Find the exact length of the curve defined by the set of parametric equations $x = t^2 + 1$, $y = t^3$ for $0 \le t \le 1$.

Answers

1. (a) diverges
   (b) converges absolutely
   (c) cannot draw conclusion
   (d) converges absolutely
2. 0
3. $\frac{5}{8}$
4. converges absolutely
5. converges absolutely
6. converges absolutely
7. $(-\infty, \infty)$, ∞
8. (-1,1), 1
9. (-1,3), 2

10. $(-\frac{1}{5}, \frac{1}{5})$, $\frac{1}{5}$

11. (-3,3), 3

12. $\left\{-\frac{1}{3}\right\}$, 0

13. $\sum_{n=0}^{\infty} \frac{3x^n}{4^{n+1}}$

14. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+2}}{2^{n+1}}$

15. $\sum_{n=0}^{\infty} \frac{x^{n+3}}{4^{n+1}}$

16. $\sum_{n=0}^{\infty} (2x)^{n+1}$

17. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}$ for $|x| < 1$

18. $\sum_{n=0}^{\infty} \frac{-x^{n+1}}{n+1}$ for $|x| < 1$

19. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$ for $|x| < 1$

20. $1 - 3(x-1) + 6(x-1)^2$

21. $1 - 2x + 3x^2$

22. $1 + 2x + 3x^2$

23. $5 - 2x + x^2$

24. $-x + 2x^2 + x^3$

25. not possible with given information

26. $x^2 - \frac{x^6}{6} + \frac{x^{10}}{120}$

27. $3x^2 - \frac{3^3 x^4}{3!} + \frac{3^5 x^6}{5!}$

28. $\sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n} x^{2n+1}}{27(2n)!}$

29. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2^{2n}(2n)!}$

30. $\displaystyle\sum_{n=0}^{\infty} \frac{3^n x^{n+2}}{n!}$

31. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}$

32. $\rho_{20} + \rho_{20}\alpha(t-20) + \dfrac{\rho_{20}\alpha^2}{2}(t-20)^2$

33. $y = (x+1)^2$

34. $x^2 + y^2 = 25$

35. $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$

36. $(x-2)^2 + (y-5)^2 = 9$

37. ellipse centered at origin with x-radius of 5 and y-radius of 2, starts and stops at (5,0), around once counterclockwise

38. semicircle centered at (2,3) with radius of 1, starts at (3,3) and stops at (1,3), counterclockwise

39. (a) $\dfrac{1}{3}x + 3$

    (b) $t = -\sqrt{\dfrac{3}{2}}$; $t = \sqrt{\dfrac{3}{2}}$

40. (a) $\dfrac{8}{9}x + \dfrac{17}{9}$

    (b) $t = \dfrac{1}{9}$

41. (a) $\dfrac{dy}{dx} = \dfrac{v_0 \sin\alpha - gt}{v_0 \cos\alpha}$

    (b) $t \approx 10.2$ seconds

42. $\dfrac{5\pi}{4}$

43. $\dfrac{1}{27}\left(13^{3/2} - 8\right)$