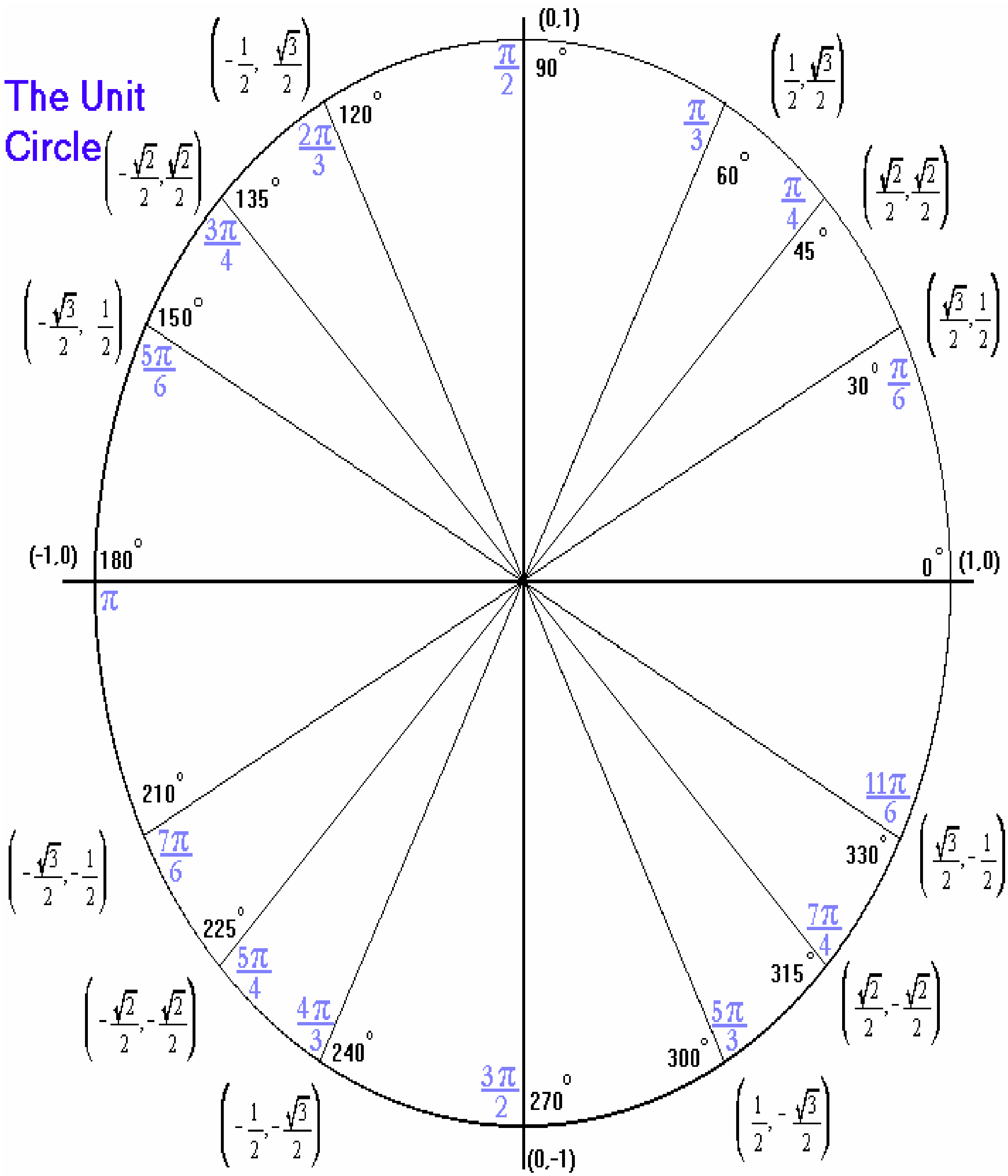
FORMULAE THAT WILL BE ON THE FINAL EXAM

FINANCE & TRIGONOMETRY FORMULAS



$$\cos^2(x)+\sin^2(x)=1 \qquad 1+\tan^2(x)=\sec^2(x) \qquad \cot^2(x)+1=\csc^2(x)$$

$$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$$
$$\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$$
$$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$$

$$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$$
$$\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$$
$$\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$$

$$\sin(2x)=2\sin(x)\cos(x)$$
$$\cos(2x)=\begin{cases}\cos^2(x)-\sin^2(x)\\ 2\cos^2(x)-1\\ 1-2\sin^2(x)\end{cases}$$
$$\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$$

$$\cos\left(\frac{x}{2}\right)=\pm\sqrt{\frac{1+\cos(x)}{2}}$$
$$\sin\left(\frac{x}{2}\right)=\pm\sqrt{\frac{1-\cos(x)}{2}}$$
$$\tan\left(\frac{x}{2}\right)=\pm\sqrt{\frac{1-\cos(x)}{1+\cos(x)}}$$

$$c^2=a^2+b^2-2ab\cos(C)$$
$$\frac{\sin(A)}{a}=\frac{\sin(B)}{b}=\frac{\sin(C)}{c}$$

$$A=P\left(1+\frac{r}{n}\right)^{nt}$$
$$A=Pe^{rt}$$

$$\mathbf{u}\cdot\mathbf{v}=\|\mathbf{u}\|\,\|\mathbf{v}\|\cos(\theta)$$

MAT170 – FINAL EXAM – REVIEW

PART I – ALGEBRA

A. *Difference quotient - Section 1.3*

Find and simplify the difference quotient, $\frac{f(x+h)-f(x)}{h}, h\neq 0$, of each of the following functions:

1) $f(x)=\frac{1}{3x}$
2) $f(x)=-x^2+5x+9$
3) $f(x)=3x^2+4x-8$

B. *Function composition - Section 1.7*

1. Find $(f\circ g)(x)$ and $(g\circ f)(x)$, where $f(x)=x^2-x+4$ and $g(x)=2x-3$.
2. Find $(g\circ f)(x)$, where $f(x)=e^{2x}-1$, and $g(x)=\ln(x+1)$.

C. *Inverse functions - Section 1.8*

Find the inverse of each of the following functions:

1) $f(x)=\frac{4x}{3+x}$
2) $f(x)=\ln(3x-2)-4$
3) $f(x)=4+e^{5x-2}$
4) $f(x)=x^3-10$

D. *Zeros of polynomials - Section 2.3-2.5*
Find the zeros of each of the following functions:

1) $f(x) = -x^3 + x^2 + 2x$
2) $f(x) = x^3 - x^2 + 9x - 9$

E. *Domains of function s- Sections 1.2-1.3*
Find the domain of each of the following functions:

1) $f(x) = \dfrac{1-x}{x^2-9}$
2) $f(x) = \dfrac{x+1}{3x-2}$
3) $f(x) = \log(2x+1)$
4) $f(x) = \sqrt{8-2x}$

F. *Exponential equations – Section 3.4*
Solve each of the following exponential equations. Give the answer, first in terms of natural logarithms, and then a decimal approximation to two decimal places:

1) $3^{2x} - 3^x - 42 = 0$
2) $5^x = 3^{x-1}$
3) $e^{2x} - 7e^x - 18 = 0$

G. *Logarithmic equations – Section 3.4*
Solve each of the following logarithmic equations.

1) $\log_2(x) + \log_2(x-7) = 3.$
2) $\log_2(3x-1) = 5$
3) $\ln(x) - \ln(x-2) = 1$

H. *Application of rational functions – Section 2.6*

1) Suppose that an insect population in millions is modeled by $f(x) = \dfrac{10x+1}{0.2x+1}$, where $x \geq 0$ is in months. What happens to the insect population after a long time?

2) A company that manufactures calculators has determined that the average cost for producing x calculators is $\overline{C}(x) = \dfrac{15000 + 20x}{x}$ dollars. In the long run, what value does the average cost approach?

I. *Application of quadratic functions – Section 2.2*

1) An astronaut on the Moon throws a baseball upward. The height of the ball is approximated by the function $h(t) = -2.7t^2 + 30t + 6.5$ feet, t is the time in seconds after the ball was thrown. When does the baseball reach its maximum height? What is the maximum height of the baseball?

2) The percent increase for in-state tuition at Arizona public universities during the years 1990 – 2002 can be modeled by $f(x) = 0.156x^2 - 2.05x + 10.2$, where x=0 represents 1990. What was the minimum percentage increase in tuition?

J. *Application of exponential functions – Section 3.4*

1) How long will it take for \$5000 to grow to \$8400 at an interest rate of 6% compounded a) quarterly? b) monthly? c) continuously?
2) Find the interest rate needed for \$3000 to accumulate to \$10000 in an account where the interest is compounded monthly for 7 years.

K. *Transformations – Section 1.6*

1) Find the function g(x) after all of the following transformations are applied to x^2.

a. Reflect about the x-axis
b. Shift left 5 units
c. Shift up 3 units

PART II – TRIGONOMETRY

A. *Domain, ranges, and graph of trig functions – Sections 4.5-4.6*

1) What is the domain and range of $\cos(x)$?
2) What is the domain and range of $\csc(x)$?
3) What is the domain and range of $\tan(x)$?
4) Suppose $y = -3\cos(2x + \pi)$; Find the amplitude, period and phase shift.

B. *Pythagorean identity, summation, difference, double angle, half-angle formula problems – Sections 5.1-5.3*

Given $\sin(\alpha) = -\frac{3}{8}$, $\pi < \alpha < \frac{3\pi}{2}$ and $\cos(\beta) = \frac{3}{5}$, $0 < \beta < \frac{\pi}{2}$ find the following

1) $\cos(\alpha)$ 2) $\sec(\alpha)$ 3) $\tan(\alpha)$ 4) $\cot(\alpha)$ 5) $\csc(\alpha)$

6) $\cos(2\alpha)$ 7) $\sin(2\alpha)$ 8) $\tan(2\alpha)$ 9) $\cos\left(\frac{\alpha}{2}\right)$ 10) $\sin\left(\frac{\alpha}{2}\right)$

11) $\sin(\alpha - \beta)$ 12) $\cos(\alpha + \beta)$

C. *Inverse trigonometric functions – Section 4.7*

1) Find an algebraic expression for $\sin(\cos^{-1}(3x))$, where $3x$ is positive and in the domain of the given inverse function.
2) Find an algebraic expression for $\cos(\tan^{-1}(x))$, where x is positive and in the domain of the given inverse function.

D. *Trig identities – Section 5.1*

Verify the trigonometric identities.

1) $\cos(x)\cot(x)+\sin(x)=\csc(x)$

3) $\sin(2x)\sin(x) + 2\cos^3(x) = 2\cos(x)$

2) $\dfrac{\cos(x)+\sin(x)-\sin^3(x)}{\sin(x)}=\cot(x)+\cos^2(x)$

E. *Trig equations – Section 5.5*

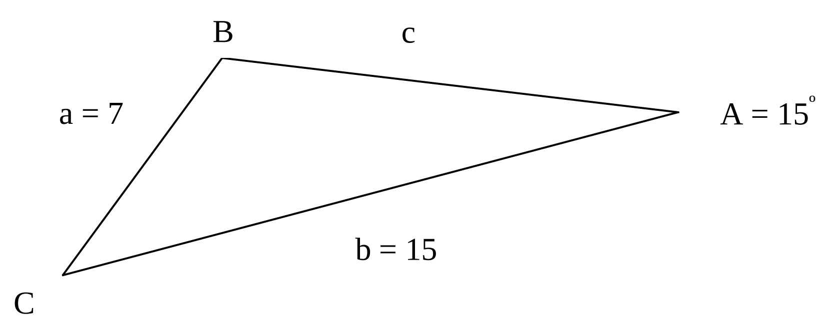
1) Solve $\sin(2x)+\sqrt{2}\cos x=0$ in the interval $[0,2\pi)$.
2) Solve $2\sin^2 x-5\sin x+2=0$ in the interval $[0,2\pi)$.

F. *Arc length – Section 4.1*

1. Algiers, Algeria and Barcelona, Spain lie on the same line of longitude. Algiers is located at 36.8°N latitude and Barcelona is located at 41.4° N latitude. Find the distance between Algiers and Barcelona. Note the radius of the earth is approximately 3960 miles.

G. *Law of sines – Section 6.1*

1) Solve the following triangle:



2) An aircraft is spotted by two observers who are 5000 meters apart. As the airplane passes over the line joining the observers, each observer takes a sighting of the angle of elevation of the airplane. The first observer sights the plane at 40° and the second observer sights the plane at 35°. How far away is the airplane from the first observer?

H. *Law of cosines – Section 6.2*

1)A pilot is flying from Jackson, Michigan, to Chicago, Illinois, a distance of approximately 200 miles. As he leaves Jackson, he flies 20 degrees off course for 50 miles. How far is he then from Chicago?

2) A tourist stands 100 ft from the base of the Leaning Tower of Pisa. With the tower leaning away from the observer, the observer looking up at an angle of 52° finds that the distance from the top of the tower to where he is standing is 228 ft. Find the angle the Leaning Tower makes with the ground?

I. *Vectors and Dot Product – Sections 6.1-6.2*

1) **v** = 5**i** – 4**j**, **w** = 2**i** – **j**

- **a) 2v + 3w =**
- **b) v – 5w =**

2) **v = 6i + j, w =** 2**i** –3**j**

- **a) v·w =**
- **b) ||v||=**
- **c) || 2w ||=**
- d) What is the angle between **v** and **w**?

ANSWERS

(Some of these answers have the possibility of not being correct)

PART I – ALGEBRA ANSWERS

A. 1) $-\dfrac{1}{3x(x+h)}$ 2) $-2x+5-h$ 3) $6x+4+3h$

B. 1) $(f\circ g)(x)=4x^2-14x+16,\ (g\circ f)(x)=2x^2-2x+5$ 2) $(g\circ f)(x)=2x$

C. 1) $f^{-1}(x)=\dfrac{3x}{4-x}$ 2) $f^{-1}(x)=\dfrac{e^{x+4}+2}{3}$

3) $f^{-1}(x)=\dfrac{\ln(x-4)+2}{5}$ 4) $f^{-1}(x)=\sqrt[3]{x+10}$

D. 1) zeros are 2, 0, –1 2) zeros are 1, 3*i*, –3*i*

E. 1) $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$ 2) $(-\infty,\frac{2}{3})\cup(\frac{2}{3},\infty)$

3.) $\left(-\frac{1}{2},\infty\right)$ 4) $(-\infty,4]$

F. 1) $x=\log_3(7)=\frac{\ln(7)}{\ln(3)}\approx 1.77$ 2) $x=\frac{\ln(3)}{\ln(3)-\ln(5)}\approx -2.15$ 3) $x=\ln(9)\approx 2.197$

G. 1) $x=8$ 2) $x=11$ 3) $x=\frac{2e}{e-1}\approx 3.16$

H. 1) The insect population approaches the size of 50 million.
2) In the long run the average cost approaches 20 dollars.

I. 1) Maximum time is $5\frac{5}{9}$ seconds. Maximum height is $89\frac{5}{6}$ feet.
2) The minimum percentage increase in tuition was approximately 3.47% when *x* = 6.57.

J. 1) a) 8.71 years b) 8.67 years c) 8.65 years
2) 17.3%

K. 1) $g(x)=-(x+5)^2+3$

PART II – TRIGONOMETRY ANSWERS

A. 1) domain $(-\infty,\infty)$; range [–1, 1]
2) domain $\{x|x\neq n\pi\}$ where *n* is an integer. ; range $(-\infty,-1]\cup[1,\infty)$
3) domain $\{x|x\neq \frac{\pi}{2}+n\pi\}$ where *n* is an integer. ; range $(-\infty,\infty)$
4) Aplitute = 3, Period = π, Phase shift = $-\frac{\pi}{2}$

B. 1) $-\frac{\sqrt{55}}{8}$ 2) $-\frac{8\sqrt{55}}{55}$ 3) $\frac{3\sqrt{55}}{55}$ 4) $\frac{\sqrt{55}}{3}$ 5) $-\frac{8}{3}$

6) $\frac{23}{32}$ 7) $\frac{3\sqrt{55}}{32}$ 8) $\frac{3\sqrt{55}}{23}$ 9) $-\frac{\sqrt{8-\sqrt{55}}}{4}$ 10) $\frac{\sqrt{8+\sqrt{55}}}{4}$

11) $\frac{-9+4\sqrt{55}}{40}$ 12) $\frac{-3\sqrt{55}+12}{40}$

C. 1) $\sqrt{1-9x^2}$ 2) $\dfrac{1}{\sqrt{1+x^2}}$

D. methods vary

E. 1) $\dfrac{\pi}{2},\dfrac{3\pi}{2},\dfrac{5\pi}{4},\dfrac{7\pi}{4}$ 2) $\dfrac{\pi}{6},\dfrac{5\pi}{6}$

F. 1) 318 miles

G. 1) $B_1 \approx 33.68°$ $C_1 \approx 131.32°$ $c_1 \approx 20.31$
$B_2 \approx 146.32°$ $C_2 \approx 18.68°$ $c_2 \approx 8.66$
2) 2969.05 meter

H. 1) distance ≈ 153.968 mile
2) 102.7°

I. 1) a) 16**i** – 11**j** b) –5**i** + **j**

2) a) 9 b) $\sqrt{37}$ c) $\sqrt{52}$ d) $\theta = \cos^{-1}\left(\dfrac{9}{\sqrt{37}\sqrt{13}}\right) \approx 65.8^\circ$