K-theory for C^*-algebras

Course   MAT 598
Semester   Spring 2020
Line No.   26725
Time   TuTh 1:30 - 2:45 PM
Room   WXLR A304
Instructor Jack Spielberg

C^*-algebras are the subject of a branch of functional analysis that is deeply entwined with many other areas of mathematics (such as algebraic topology, geometry, number theory, index theory). The most immediate point of contact is with topology, since locally compact Hausdorff spaces are essentially the same thing as commutative C^*-algebras. K-theory originated in the problem of classifying vector bundles over compact Hausdorff spaces (in particular, over manifolds). When these ideas are translated to algebraic form (i.e. to commutative C^*-algebras), it turns out that the theory applies in an extremely natural way to noncommutative C^*-algebras as well. This gives a huge expansion of the theory, far beyond its origins. K-theory has become a fundamental tool in the study of C^*-algebras. In this course we will work through the construction of the K-groups and their properties, and will consider various examples along the way.

The essential prerequisites are some familiarity with the basic notions of C^*-algebras. More specifically, the most basic aspects of C^*-theory have to do with the operator norm on the space of complex $n \times n$ matrices. Most examples are infinite dimensional, but some comfort with making estimates in finite dimensions will go a long way with the techniques we will use. In addition, basic facts from abstract algebra and complex analysis are unavoidable. These will be reviewed a bit as needed. The most essential course prerequisites are metric space analysis (e.g. MAT 472) and abstract linear algebra (e.g. MAT 442). I will assign homework three or four times during the semester. There will be no exams.

The lectures will be self-contained. Some references are:

1. "An introduction to K-theory for C^*-algebras" by Rørdam, Larsen, Laustsen (probably the best book for starting)
2. "K-theory" by Blackadar (extremely complete and hard to read)
3. "K-theory and C^*-algebras" by Wegge-Olsen (friendly and with lots of discussion - maybe excessively wordy, but sometimes that's nice)

Questions may be addressed to the instructor at spielberg@asu.edu.