

Course Description: This course will provide a modern introduction to number theory in the setting of function fields.

Classical elementary number theory consists of studying the properties of arithmetic in \mathbb{Q} and its associated ring of integers \mathbb{Z} (e.g., prime factorization, Fermat's little theorem, quadratic reciprocity, the prime number theorem, and so forth).

Algebraic number theory then expands this discussion to algebraic extensions K of \mathbb{Q} and their associated rings of integers \mathcal{O}_K (e.g., unique factorization into prime ideals, Dirichlet's unit theorem, zeta functions and L -functions, and so forth).

The primary aim of this course is to discuss the analogous stories of elementary number theory in the setting of the function field $\mathbb{F}_q[t]$ and of algebraic number theory in finite extensions of $\mathbb{F}_q[t]$. There are a great many analogies between number theory over algebraic number fields and over algebraic function fields, and our goal is to elucidate (as much as we can) the connections between the two.

Textbook: Rosen's "Number Theory in Function Fields", and (depending on interest) additional resources and papers.

Audience/Prerequisites: This course is intended for any advanced undergraduate or graduate students interested in number theory. A degree of comfort with commutative algebra and Galois theory (at the level of the 543-544 graduate algebra sequence) is highly recommended. For undergraduates, the background would correspond to 441 (rings) and 444 (fields and Galois theory), along with some independent reading on modules. Other courses that would be useful to help provide some context include 445 (elementary number theory), 542 (elliptic curves) and 547 (algebraic number theory). We will use some basic algebraic geometry, but the material will be self-contained.

Course Topics: We will begin with a treatment of elementary number theory in $\mathbb{F}_q[x]$, studying primes and factorization, zeta functions, the power reciprocity law, and the analogue of Dirichlet's theorem on primes in arithmetic progression. We will then discuss algebraic function fields, differentials and divisors, Galois theory of function fields, class groups, cyclotomic function fields, L -series, S -units, and other related topics as interest dictates.

Presentations: Students will be expected to give an in-class presentation during the semester on some topic related to the course. The presentation may be on a topic related to the course, or provide some additional perspective from a related area. (Students who have previously taken algebraic number theory are especially encouraged to present the algebraic number theory side of some of the course material.) An option to solve and then present the solutions to one or several homework-style problems will also be available.