MAT 275 Course Objectives

Students will be able to:

- Sketch (BY HAND) a direction field for a first-order ODE, and sketch integral curves.

- Find equilibrium (constant) solutions of ODEs of the form $y' = f(y)$ and classify them as stable, unstable or semi-stable.

- Read the long-term behavior of the solutions of an autonomous first-order differential equation for different initial conditions given its direction field.

- Verify by substitution that a given function is a solution of a given ODE.

- Use initial conditions to find the values of the constants in a given general solution to an ODE.

- Classify differential equations by their order and linearity.

- Derive differential equations that model simple applied problems.

- Solve applied problems such as mixture problems and problems involving Newton's Law of Cooling.

- Use the method of integrating factors to solve linear, first-order ODEs.

- Solve first-order separable ODEs and initial value problems (IVPs). Whenever possible , write the solution in explicit form and determine the largest interval in which the solution is defined.

- Understand and apply the theorems on existence and uniqueness of solutions to initial value problems for first-order ODEs.

- Use Euler's method to derive approximations of solutions to initial value problems for first-order ODEs.

- Determine whether two functions of one variable are linearly independent or dependent on a given interval by determining whether one of them is a constant multiple of the other or not.

- Classify second-order linear differential equation in standard form including distinguishing between homogeneous and nonhomogeneous equations.

- Apply the superposition principal for linear, homogeneous ODEs.

- Find the Wronskian of a set of functions as well as fundamental sets of solutions for linear, homogeneous ODEs with constant coefficients.

• Find general solutions to linear, homogeneous ODEs with constant coefficients using the roots of the characteristic polynomial.

• Solve nonhomogeneous, linear ODEs using the method of undetermined coefficients. (A method that works only when the function $g(t)$ is a polynomial, an exponential function, a sine or cosine and and/or a sum/product of these functions.)

• Apply ODEs to model mechanical and electrical oscillations.

• Transform a linear ODE of any order n into a system of n first-order ODEs.

• Write a system of first-order linear ODEs in matrix form.

• Solve simple 2-dimensional linear systems of ODEs by rewriting them as a single higher-order ODEs.

• Determine the phase portraits of simple 2-dimensional systems

• Know the definition of the Laplace Transform.

• Calculate the Laplace Transform of basic functions using the definition.

• Find the Laplace transform of derivatives and antiderivatives of functions.

• Compute inverse Laplace Transforms.

• Apply Laplace Transforms to find solutions of initial value problems for linear ODEs.

• Write piecewise functions in terms of unit step functions and find their Laplace Transforms.

• Solve certain ODEs where the forcing term is given by a piecewise continuous function.

• Learn the definition of the Dirac delta generalized function, understand it as an impulse, and solve ODEs with forcing terms involving impulses.

• Understand the definition and concept of linearly independent and linearly dependent vectors.

• Find the determinant, eigenvalues and eigenvectors of a given matrix.

• Write a linear system of ODEs in the form $x' = P(t)x + f(t)$ with defining explicitly the coefficient the coefficient matrix $P(t)$ and the vectors x of unknowns and $f(t)$ of forcing terms.

• Verify by substitution that a given vector function is a solution to a given system of linear ODEs.

• Find the Wronskian of 2 given vector functions $x_1(t), x_2(t)$. Understand and explain what it means for it to be nonzero.

• Use the eigenvalue/eigenvector method to solve systems linear, homogeneous ODEs with constant coefficients $x' = Ax$ where the $n \times n$ coefficient matrix A is real-valued and non-defective ($n \leq 3$).